JOURNAL OF PROPULSION AND POWER
Vol. 17, No. 4, July–August 2001

Evaporation of Sprays of Liquid Fuels and Liquid Oxidizers in a Shear Layer

D. Katoshevski*
Ben-Gurion University of the Negev, 84105 Beer-Sheva, Israel
and
Y. Tambour†
Technion—Israel Institute of Technology, 32000 Haifa, Israel

In rocket engines propelled by liquid fuels and liquid oxidizers, one may find regions where two adjacent spray streams, one that carries fuel droplets and the other that carries the droplets of the liquid oxidizer, travel at different velocities and form a shear flow. A theoretical study of such unidirectional multisize (polydisperse) evaporating spray streams is presented. In each of the streams droplets of different sizes are assumed to travel at different longitudinal velocities, that is, drop-size correlated velocities are assumed for both the liquid fuel and the liquid oxidizer. The lateral evolution in drop-size distributions across the shear layer is analyzed, and its effect on the lateral spread of the fuel vapor and the vapors produced by the droplets of the liquid oxidizer is examined. It is shown that the polydisperse drop-size distributions of the liquid fuel as well as that of the liquid oxidizer control the vapor production rates and determine the shape of the vapor concentration profiles across the shear layer.

Nomenclature

$\hat{A}$	=	characteristic evaporation rate
$\bar{B}^i_{j,j+1}$	=	sectional vaporization coefficient of spray i
$\bar{C}^i_j$	=	sectional vaporization coefficient of spray i
c_p	=	heat capacity at constant pressure
D_j	=	sectional diffusion coefficient
d_j	=	upper range of diameter at section j
E^i_j	=	sectional vapor source term of spray i
f	=	function of η
h_v	=	latent heat of vaporization
N_s	=	total number of droplet-size sections
Pr	=	Prandtl number
p	=	normalized pressure
Q^i_j	=	normalized mass fraction of size section j of spray i
$\hat{Q}_{t,c}$	=	characteristic total mass fraction of the liquid phase
$q_j(x)$	=	ratio of the velocity of droplets from section j and the host gas velocity
Re	=	Reynolds number
Sc	=	Schmidt number
$\bar{T}$	=	normalized temperature
$\hat{T}_c$	=	characteristic temperature
$\hat{U}$	=	characteristic longitudinal velocity
u	=	longitudinal velocity of the gas flow
u_j	=	longitudinal velocity of the droplets in size section j
v	=	lateral velocity of the gas flow
v_j	=	lateral velocity of the droplets in size section j
x	=	normalized longitudinal coordinate
$\hat{x}_c$	=	characteristic longitudinal distance
y	=	normalized lateral coordinate
α	=	power of x in the expression for the outer velocity U
β	=	parameter related to the power α
Δ^i_v	=	Damkohler-like number for evaporation, $\hat{A}^i \hat{x}_c / \hat{U}$
ζ^i_j	=	source term, represents the effect of the velocity lag between the droplets and host gas
η	=	similarity variable
μ	=	host gas viscosity
$\bar{\nu}$	=	kinematic viscosity
ρ	=	host gas density
ψ	=	stream function

Subscripts

c	=	characteristic value
I	=	upper stream
II	=	lower stream
j	=	section number

Superscripts

f	=	fuel
i	=	kind of spray, $i = f, o$
o	=	oxidizer

Received 13 June 2000; revision received 18 January 2001; accepted for publication 29 January 2001.

*Senior Lecturer, Member of the Israeli Society of Aerospace Sciences, Department of Environmental Engineering.

†Stephan Berger Professor of Aeronautical Engineering, Department of Aerospace Engineering.

Introduction

MORE than three decades ago, many combustion researchers foresaw the importance of the study of liquid droplet combustion. The classical analytical analyses by Spalding[1] and by Godsave[2] and early experimental studies, such as by Krier and Wronkiewicz,[3] emphasized the importance of liquid fuel combustion for propellant rocket motors. As Kuo points out (see Ref. 4, page 518), when only a few injectors (of liquid fuel, and sometimes also of liquid oxidizer) are used in a rocket engine motor, it is important to analyze the shear layer flowfield near the injectors and the mixing of the fuel vapor and the vapors produced by the oxidizer. The analysis of streams of evaporating sprays of liquid fuels and liquid oxidizers is thus one of the most important issues in the design of rocket engines propelled by liquid fuels and liquid oxidizers. In the near-field close to the injectors of the fuel and the oxidizer, one may find regions where two adjacent spray streams of different velocities produce vapors that spread in the lateral direction of a shear layer that is formed.

Sprays produced by industrial atomizers are usually comprised of drops of a wide range of sizes, which travel downstream at different velocities, which are in most cases drop-size dependent (see Chiu and Chigier[5] and Presser et al.[6]). Such sprays are termed multisize or polydisperse sprays. Theoretical studies of evaporating polydisperse sprays in shear layer flows were presented by the authors[7–9] with and without the presence of a flame. For a nonburning spray, the qualitative comparison between the results of Katoshevski and Tambour[7] for typical computed total mass distributions of the liquid-phase and experimental data reported by Lazaro and Lasheras[10] shows a strong similarity between the two sets of profiles. This supports the assumptions and boundary conditions employed in Katoshevski and Tambour's theoretical study.[7] The general behavior of the theoretical Sauter mean diameter (SMD) distribution of the spray across the shear layer also compared well with the reported experimental results of Lazaro and Lasheras.[10]

In spray combustion research, it is important to analyze the evaporation of the fuel droplets on the one hand and the evaporation of the droplets of the liquid oxidizer on the other hand because the lateral spread of the fuel vapor and the vapors of the liquid oxidizer eventually determine the location and structure of the diffusion flame that will result on ignition.

The case of a polydisperse burning fuel spray in a shear layer flow in which a gaseous oxidizer is present in the coflowing stream was studied by Tambour and Katoshevski.[9] The authors analyzed the effects of fuel drop-size distributions on flame location and temperature. They showed that sprays that contain mainly small droplets result in a flame geometry that is flatter than the geometry of flames formed by sprays that contain mainly large droplets. In addition, their computed results indicated that under certain operating conditions sprays of different initial drop-size histograms, but of similar SMD values, form flames of similar shapes.

The study of a diffusion flame fed by vapors that are produced by sprays of a liquid fuel and a liquid oxidizer was recently carried out by the authors.[11] However the main emphasis in the study was on the analysis of the flame, whereas the purpose of the present study is to focus on the behavior of evaporating multisize polydisperse sprays of liquid fuels and liquid oxidizers and their effect on fuel and oxidizer vapor production rates, which determine the lateral spread and concentration of these vapors across the shear layer.

Spray Equations for the Liquid Fuel and the Liquid Oxidizer

Each spray is assumed to be of a multisize (polydisperse) nature, where the initial drop-size histograms for the liquid fuel and for the spray of the liquid oxidizer are given in Table 1. It is assumed that the streams of the fuel and the oxidizers travel at different velocities and form a steady incompressible laminar shear layer flow (see Fig. 1). The upper stream, denoted by I, carries the multisize spray of fuel droplets and the other stream, II, carries a multisize spray of droplets of the liquid oxidizer. The longitudinal and lateral directions are x and y, respectively, and we distinguish here between the longitudinal freestream velocity of the fuel stream U_I and that of the oxidizer stream U_{II}

$$U_I = \hat{U}_I (x/\hat{x}_c)^\alpha \tag{1}$$

$$U_{II} = \hat{U}_{II} (x/\hat{x}_c)^\alpha \tag{2}$$

where $\hat{x}_c$ is a characteristic longitudinal distance and $\hat{U}_I$ and $\hat{U}_{II}$ are characteristic longitudinal velocities for streams I and II. For constant-velocity or decelerating flows, the power α is zero or negative.[7,8]

For later use, we omit the index I from the notation for the characteristic velocity of the upper stream, that is,

$$\hat{U} \equiv \hat{U}_I \tag{3}$$

and define the Reynolds numbers for streams I and II as

$$Re_I = \hat{U}\hat{x}_c/\bar{\nu}_I \tag{4a}$$

$$Re_{II} = \hat{U}\hat{x}_c/\bar{\nu}_{II} \tag{4b}$$

Using the conventional boundary-layer normalization of the gas phase continuity and momentum equations one obtains

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = S_v \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{\hat{x}_c}{\hat{U}^2}\frac{1}{\rho}F_x \tag{6}$$

$$\frac{\partial p}{\partial y} = 0 \tag{7}$$

where u and v are the normalized gas velocity components in the x and y directions, respectively. S_v is a source term due to evaporation and p is the normalized pressure.[7] F_x describes the interaction between the gas and droplets and is important in the near field where a velocity lag exists. This issue will be addressed here later.

Because the pressure does not change in the lateral direction [Eq. (7)], the pressure gradient in Eq. (6), can be replaced by the term $-U\mathrm{d}U/\mathrm{d}x$, which is the longitudinal pressure gradient outside the shear layer.

For describing the spray dynamics, we use the sectional method that was employed by Katoshevski and Tambour[7–9] and is extended here for a two-spray system,

$$u_j\frac{\partial Q_j^i}{\partial x} + v_j\frac{\partial Q_j^i}{\partial y} = \frac{D_j}{\bar{\nu}}\frac{\partial^2 Q_j^i}{\partial y^2} + \frac{\hat{x}_c}{\hat{U}}\left(-C_j^i Q_j^i + B_{j,j+1}^i Q_{j+1}^i\right)$$
$$j = 1, 2, \ldots, N_s, \qquad i = f, o \tag{8}$$

where j denotes a droplet size section and i is for distinguishing between the two kinds of sprays, f fuel and o oxidizer. N_s is the total number of sections, where the same number of sections is used for each of the sprays. D_j is a sectional diffusion coefficient that represents lateral transport of droplets across the shear layer due to drag forces and vortices.[7] u_j and v_j are the droplet velocities. In the near field the droplets are either decelerating or in the process of catching up with the host gas, and thus, their velocities are different from the corresponding velocities u and v of the host gas. The velocity lag of the droplets depends on their size and is represented by an approximate relation,

$$U_j^i = q_j^i(x)U^i \tag{9}$$

where the function $q_j(x)$ is the ratio of the velocity of droplets' from section j at the edge of the shear layer and the gas stream velocity at each edge. At the far field (far in the longitudinal distance from the atomizer), the preceding ratio converges to unity for both sprays, as the droplets reach the host gas velocity.

The function $q_j(x)$ could be described by using the droplets equation of motion in the outer-flow region.[7] Within the shear layer, we

Table 1 Histograms and spray properties of the liquid fuel and the liquid oxidizer

	Section									
Parameter	1	2	3	4	5	6	7	8	9	10
$d_l - d_u$, μm	0–3	3–6	6–9	9–12	12–15	15–18	18–21	21–24	24–27	27–30
Size–velocity q_j	0.4	0.42	0.45	0.53	0.60	0.66	0.73	0.80	0.86	0.93
Correlation										
$\bar{C}_j$	120.0	22.9	10.6	6.5	4.6	3.5	2.8	2.4	2.0	1.8
$\bar{B}_{j,j+1}$	5.7	4.2	3.3	2.6	2.2	1.9	1.7	1.4	1.4	
Fuel histogram	0.07	0.10	0.13	0.17	0.17	0.12	0.09	0.06	0.06	0.03
Oxidizer histogram	0.05	0.52	0.31	0.08	0.04	0	0	0	0	0
Flow	$\alpha = 0$;	$U_{II}/U_I = \frac{1}{2}$								
Temperature correlation power		$\lambda = 2.7549$								

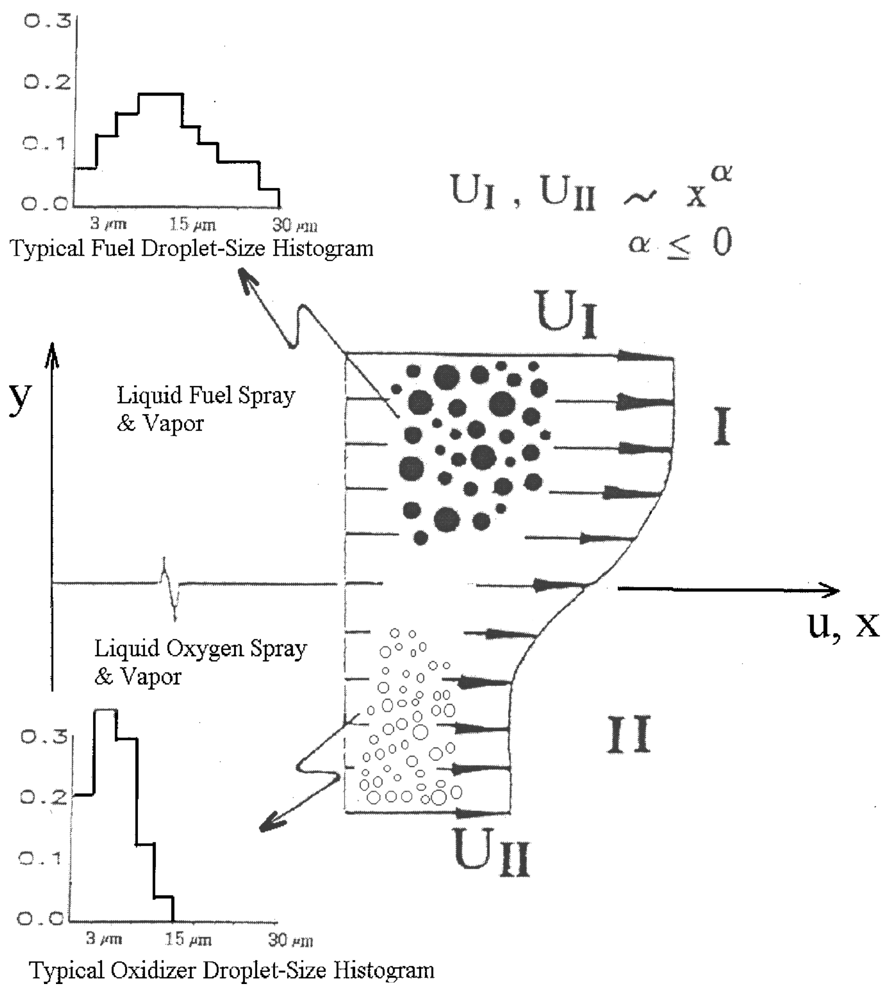

Fig. 1 Schematic description of the problem: stream *I*, polydisperse spray of liquid fuel; and stream *II*, polydisperse spray of liquid oxidizer.

assume that the velocity lag of droplets in the longitudinal direction can be described in a similar way, that is,

$$u_j^i = q_j^i(x)u \tag{10}$$

whereas in the lateral direction, we write

$$v_j^i = v \tag{11}$$

and in the droplet-diffusion coefficient D_j, we incorporate the lateral transport of droplets due to drag forces and vortices.

The diffusion equations for the fuel vapor and for the oxidizer and the energy equation in terms of temperature will be presented later in their final form after employing the following similarity transformation.

Similarity Transformation

To obtain similarity solutions for the various conservation equations considered here, we first define a stream function ψ (see also Ref. 7) and a similarity variable η

$$\Psi_I = \left[\tfrac{1}{2}(\alpha+1)\right]^{-\frac{1}{2}} x^{(\alpha+1)/2} f_I(\eta_I) \tag{12}$$

$$\eta_I = \left[\frac{1}{2}(\alpha+1)\right]^{\frac{1}{2}} x^{(\alpha-1)/2} \int_0^{y_I} \frac{\rho}{\rho_{I,\infty}}\,\mathrm{d}y_I \tag{13}$$

where I implies that these equations are for the upper stream. The same equations but with an index II are for the lower stream. For the initial investigation, density changes are not considered (with this respect, see Matkowsky and Sivashinsky[12]).

The velocity components u and v are obtained next from the derivatives of the stream function, in terms of the similarity variable, as

$$u_I/U_I = f_I' \tag{14}$$

and

$$v_I = -[(\alpha+1)/2]^{-\frac{1}{2}} x^{(\alpha-1)/2}\{[(\alpha+1)/2]f_I + [(\alpha-1)/2]\eta_I f_I'\} \tag{15}$$

where the prime denotes differentiation with respect to η, that is, $f' = \mathrm{d}f/\mathrm{d}\eta$, and here again the same I index is replaced by II for the lower stream.

Substituting Eqs. (14) and (15) into the gas longitudinal momentum equation yields the following upper stream and lower stream momentum equations:

$$f_I''' + f_I f_I'' + \beta\left\lfloor 1 - (f_I')^2 \right\rfloor = 0 \tag{16}$$

$$f_{II}''' + f_{II} f_{II}'' + \beta\left[(U_{II}/U_I)^2 - (f_{II}')^2\right] = 0 \tag{17}$$

where $\beta = 2\alpha/(\alpha+1)$.

For the upper and lower edges of the shear layer, the boundary conditions are given by

$$\eta_I \to +\infty: \qquad f_I' = 1 \tag{18}$$

$$\eta_{II} \to -\infty: \qquad f_{II}' = U_{II}/U_I \tag{19}$$

and for the matching boundary conditions at the interface and other details, see Ref. 7. Note that for obtaining the momentum equations

(16) and (17) in terms of the similarity variable η only, one has to impose F_x in Eq. (6) to equal zero.

In related studies[7–9] only a spray of one kind was considered, that is, droplets were suspended only in the upper stream, which was the only stream in motion. Because all of these conditions are different here, one needs to make the proper changes in the spray and gas-phase similarity equations, as presented next. The spray equations are written as follows:

$$\left(Q_j^{i\prime\prime}/Sc_j\right) + f\,Q_j^{i\prime} = -E_j^i + \zeta_j^i \tag{20}$$

where, as in Eq. (8), i is either the oxidizer o or the fuel f and $j = 1, 2, 3, \ldots, N_S$. In the preceding equation f is the proper function (f_I for the upper stream and f_{II} for the lower stream).

In Eq. (20), the evaporation source term E_j^i and the other source term ζ_j^i, which is due to the droplet velocity lag, are given by

$$E_j^i = -(2-\beta)(1-f')\Delta_v^i\left(\bar{C}_j^i Q_j^i - \bar{B}_{j,j+1}^i Q_{j+1}^i\right) \tag{21}$$

$$\zeta_j^i = \left(1-q_j^i\right)(1-\beta)\eta\left[f' Q_j^{i\prime} + f'' Q_j^{i\prime}\right] \tag{22}$$

where Sc_j is the droplets' Schmidt number $Sc_j = \bar{\nu}/D_j$ and Δ_v^i is a Damkohler-like number for vaporization.[7] The preceding normalized evaporation coefficients $\bar{C}_j^i$ and $\bar{B}_{j,j+1}^i$ are obtained by dividing the dimensional ones [Eq. (8)] by $Ax^{\alpha-1}$, where A is a characteristic vaporization rate (see Ref. 7). These normalized evaporation coefficients are assumed to be proportional to the normalized temperature $\bar{T}$ to the power of λ, which is a correlated experimental value (Table 1).

$$\bar{C}_j^i, \bar{B}_{j,j+1}^i \sim [\bar{T}(\eta)]^\lambda R_M(Re, Pr) \tag{23}$$

In the preceding equation, R_M is the Ranz–Marshall correction term

$$R_M(Re, Pr) = 1 + aRe_j^{\frac{1}{2}} Pr^{\frac{1}{3}} \tag{24}$$

and the sectional Reynolds number Re_j incorporates the slip velocity, which is proportional to $[1 - q_j^i(x)]$, where a is a constant.

For the temperature, the fuel vapor m_f and the vapors produced by the liquid oxidizer m_o one obtains

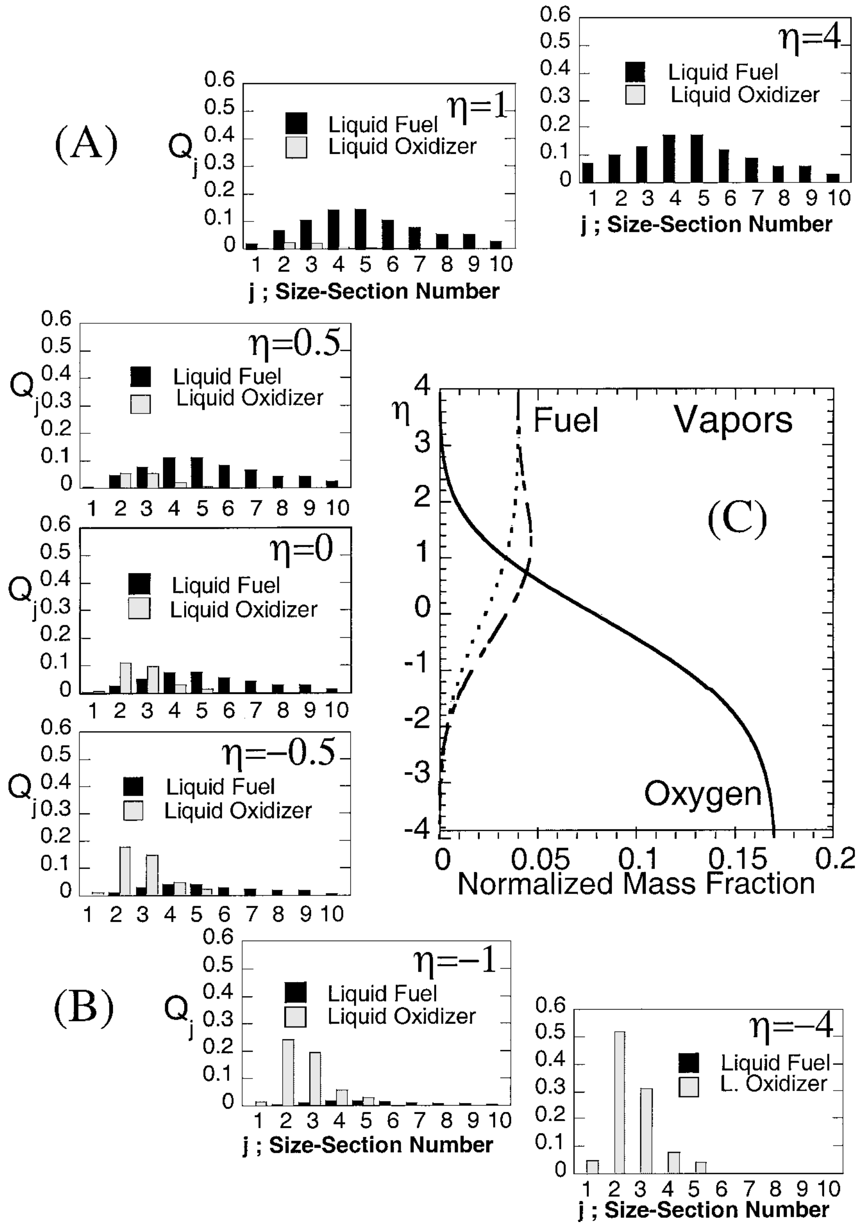

Fig. 2 a) Lateral evolution of the liquid fuel spray: histograms at stations η = +4, +1, +0.5, 0, and − 0.5; b) lateral evolution of the spray of the liquid oxidizer: histograms at stations η = − 4, − 1, − 0.5, 0, and +0.5; c) lateral profiles of the fuel vapors and vapors produced by the liquid oxidizer: · · · · ·, lower evaporation rate.

$$\frac{\bar{T}''}{Pr} + f\bar{T}' = -\frac{\hat{Q}_{t,c}h_v^f}{c_p\hat{T}_c}\left(\sum_{j=1}^{N_S} E_j^f + \frac{h_v^o}{h_v^f}\sum_{j=1}^{N_S} E_j^o\right) \quad (25)$$

$$\frac{m_f''}{Sc_f} + fm_f' = \sum_{j=1}^{N_S} E_j^f \quad (26)$$

$$\frac{m_o''}{Sc_o} + fm_o' = \sum_{j=1}^{N_S} E_j^o \quad (27)$$

In the diffusion equations (26) and (27) the right-hand-side (RHS) terms represent the fuel vapor production

$$\sum_{j=1}^{N_s} E_j^f$$

and vapor produced by the liquid oxidizer

$$\sum_{j=1}^{N_s} E_j^o$$

The RHS term in the temperature equation (25) represents the effect of the latent heat of vaporization, where h_v^f and h_v^o are the latent heats of the liquid fuel and liquid oxidizer, respectively. The temperature is normalized by $\hat{T}_c$, where the subindex c denotes a characteristic value. The preceding equations are integrated here numerically. The solutions are presented and discussed next.

Results and Discussion

We use 10 drop-size sections between 0 and 30 μm in droplet diameter to represent the multisize fuel spray. The largest droplet in the initial drop-size distribution of the liquid oxidizer is only 15 μm in diameter, and therefore, it occupies only the first five sections (see Table 1).

The evolution in drop-size histograms across the shear layer for both the liquid fuel and the spray of the liquid oxidizer are shown in Fig. 2 for various lateral stations, $\eta = +4, +1, +0.5, 0, -0.5, -1$, and -4. As one marches down from $\eta = 4$ toward the negative values of η one can clearly notice the decrease in the mass of fuel droplets as the liquid fuel evaporates (see Fig. 2a). On the other hand, as one marches up from the negative values of η toward the positive values of η, the mass of the liquid oxidizer decreases due evaporation (see Fig. 2b). The rapid evaporation of the small droplets is more pronounced in the liquid oxidizer histogram (compare histogram at station $\eta = -1$ with the one at $\eta = 0$) because the liquid oxidizer is comprised mainly of small droplets. This effect is manifested also in Fig. 3, in which the total mass profiles for the liquid fuel and the liquid oxidizer are shown. In the overlap zone, where both fuel and oxidizer droplets reside, droplet coalescence is likely to occur due to differences in drop velocities from both streams. However, this effect was not taken into account in the present study. Effects of coalescence between droplets within each of the streams were also neglected here, although we have assumed in our analysis that drops of different sizes travel at different lateral velocities (and, thus, actually may collide and coalesce).

The lateral profiles, across the shear layer, for the mass fractions of fuel vapor and for vapors produced by the liquid oxidizer are shown in Fig. 2c. These profiles are linked to the drop-size distribution histograms in which one can clearly notice the missing mass in the histograms as the droplets vaporize. These vaporizing droplets are responsible for the buildup of vapors in the vapor profile. As vapors are produced, they are convected in the longitudinal direction and diffuse laterally. However, when large amount of vapors are locally produced, the vapor profile may exhibit a local overshoot in vapor concentration, as can be seen in Fig. 2c. For lower fuel evaporation rate, representing the case where a less volatile liquid fuel is considered, see the dotted line in Fig. 2c, no local overshoot is present in the fuel vapor concentration. In the preceding case, as expected, one also finds a larger mass fraction of the remaining liquid phase, see broken line in Fig. 3.

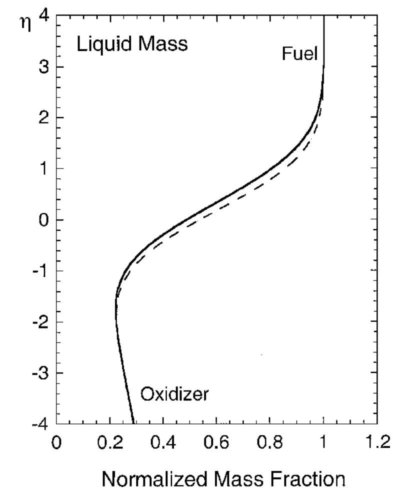

Fig. 3 Lateral distributions of the total liquid mass: broken line, lower evaporation rate.

Conclusions

It has been shown in the current theoretical work how the vapors and the liquid phase are spread across a shear layer formed by a two-spray system, where one stream carries fuel droplets and the other carries oxygen droplets. Such lateral profiles are important for combustion studies because, together with other parameters, they determine flame characteristics once ignition takes place. Thus, the study presented here sets the theoretical basis for further research in this topic as well as for flame analysis in such two-spray systems, which are common configurations in liquid rocket propulsion.

Acknowledgment

This research was supported by the fund for the promotion of research at the Technion.

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